VIRILE AGITUR

2012
TRIAL
Examination

Number: ____________________

Teacher: ____________________

Year 12 Extension 1 Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen only
- Board approved calculators may be used
- A table of standard integrals is provided at the back of the paper
- Show all necessary working in questions 11-14

Teachers:
Mr Bradford
Mr Harnwell
Mr Sedgman
Miss Yamaner*

Section I ~ Pages 1-3

- 10 marks
- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II ~ Pages 4-9

- 60 marks
- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

Write your Board of Studies Student Number on the front cover of each answer booklet

This paper MUST NOT be removed from the examination room.

Number of Students in Course: 69

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Section I

10 marks
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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1 Two dice are rolled and the sum of the numbers is written down. Find the probability of rolling a total less than 6.

(A) $\frac{1}{4}$

(B) $\frac{5}{36}$

(C) $\frac{5}{12}$

(D) $\frac{5}{18}$

2 What are the domain and range of $y = \cos^{-1}\left(\frac{5x}{2}\right)$?

(A) Domain: $-2.5 \le x \le 2.5$ and Range: $0 \le y \le \pi$

(B) Domain: $0 \le x \le \frac{5\pi}{2}$ and Range $-1 \le y \le 1$

(C) Domain: $-\frac{5\pi}{2} \le x \le \frac{5\pi}{2}$ and Range: $0 \le y \le \pi$

(D) Domain: $-\frac{2}{5} \le x \le \frac{2}{5}$ and Range: $0 \le y \le \pi$

3 How many fourteen-letter arrangements of LONDONOLYMPICS are possible**?**

(A) $\frac{14!}{12!}$

(B) $\frac{14!}{4!}$

(C) $\frac{7!}{2!3!}$

(D) $\frac{14!}{2!+2!+3!}$

4 What is the acute angle between the lines $y = 2x - 1$ and $x - 3y + 6 = 0$?

(A) $18°$
(B) $45°$
(C) $63°$
(D) $82°$

5 At a dinner party, the host, hostess and their six guests sit at a round table. In how many ways can they be arranged if the host and hostess are separated?

(A) 720
(B) 1440
(C) 3600
(D) 5040

6 If $\lim_{x \to H} \frac{2x^3 + 4x}{x^2 - 4} = \infty$ what is a value for H?

(A) $H = \infty$
(B) $H = 4$
(C) $H = 1$
(D) $H = 0$

7 If $t = \tan \frac{x}{2}$ which of the following is an expression for $\frac{dx}{dt}$?

(A) $\frac{2}{1+t^2}$
(B) $1+t^2$
(C) $\frac{1}{2}\left(1+t^2\right)$
(D) $\frac{1}{1+t^2}$

8 Which of the following is an expression for $\int \frac{2x}{\sqrt{1+x^2}}\,dx$?

(A) $\log_e(1+x^2)+C$

(B) $\log_e\sqrt{1+x^2}+C$

(C) $\sqrt{1+x^2}+C$

(D) $2\sqrt{1+x^2}+C$

9 Point A is moving on the curve $y=2x^3$ in such a way that its x-coordinate is changing at a constant rate of 0.5 units per second. What rate is the gradient changing when $x=1$?

(A) $0.5\ \text{s}^{-1}$

(B) $2\ \text{s}^{-1}$

(C) $6\ \text{s}^{-1}$

(D) $12\ \text{s}^{-1}$

10 We can express sin x and cos x in terms of tan $\frac{x}{2}$, for all values of x except…….

(A) $x=\ldots\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4}\ldots$

(B) $x=\ldots\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2}\ldots$

(C) $x=\ldots\pi,3\pi,5\pi\ldots$

(D) $x=\ldots2\pi,6\pi,8\pi\ldots$

End of Section I

Section II

60 marks
Attempt questions 11 – 14
Allow about 1 hour 45 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question.

Question 11 (15 marks) Use a SEPARATE writing booklet **Marks**

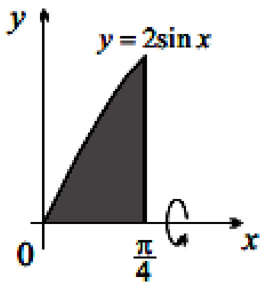
(a) Solve $\frac{5x}{x-2} \geq 3$. **3**

(b) (i) Show that the function $g(x) = x^2 - \log_e(x+1)$ has a zero between 0.7 and 0.9. **1**

(ii) Use the method of halving the interval to find an approximation to this zero of $g(x)$, correct to one decimal place. **2**

(c) Find the term independent of x in the expansion of $\left(4x^3 - \frac{1}{x}\right)^{12}$. **2**

(d)



The diagram above shows the region bounded by the curve $y = 2\sin x$, the x-axis and the line $x = \frac{\pi}{4}$. Find the exact volume of the solid generated when the shaded region is rotated about the x-axis. **2**

Question 11 continues on page 5

(e) Molten plastic at a temperature of 250º C, is poured into a mould to form a car part. After 20 minutes the plastic has cooled to 150º C. If the temperature after t minutes, is T º C, and the surrounding air temperature is 30º C, then the rate of cooling is given by:

$$\frac{dT}{dt} = -k(T-30), \text{ where } k \text{ is a constant.}$$

(i) Show that $T = 30 + Ae^{-kt}$, where A is a constant, satisfies the equation. **1**

(ii) Show that the value of A is 220º C. **1**

(iii) Find the value of k to 2 decimal places. **1**

(iv) The plastic can be taken out of the mould when the temperature drops below 80º C. How long after the plastic has been poured will the temperature be reached? Give your answer to the nearest minute. **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) A mobile phone company has a success rate of 70% when signing up new customers who enter a particular store. If 10 new customers walk into the store:

(i) Find the probability that 9 of these people sign up. Give your answer to the nearest whole percentage. **1**

(ii) What is the most likely number of customers to sign up? **1**

(b) The polynomial $P(x) = x^3 + ax + b$ has $(x-5)$ as one of its factors and has a remainder of -60 when divided by $(x+5)$. Find the values of a and b. **3**

(c) Use the substitution $u = \tan x$ to evaluate $\int_0^{\frac{\pi}{3}} \tan^2 x \sec^2 x \, dx$. **3**

(d) A particle moves in a straight line and its displacement x metres from the origin after t seconds is given by:

$$x = \cos^2 3t, \; t > 0.$$

(i) When is the particle first at $x = \dfrac{3}{4}$? **1**

(ii) In what direction is the particle travelling when it is first at $x = \dfrac{3}{4}$? **2**
Give a reason for your answer.

(iii) Express the acceleration of the particle in terms of x. **2**

(iv) Hence, show that the particle is undergoing simple harmonic motion. **1**

(v) State the period of the motion. **1**

Question 13 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) A Rotary Club has 12 females and 25 male members. The club is to choose a representative team consisting of 2 women and 4 men to send to an international conference. In how many ways can this representative team be chosen? Express your answer as an ordinary numeral. **2**

(b) The polynomial $P(x) = 2x^3 - 5x^2 + kx + 40$ has roots α, β and γ.

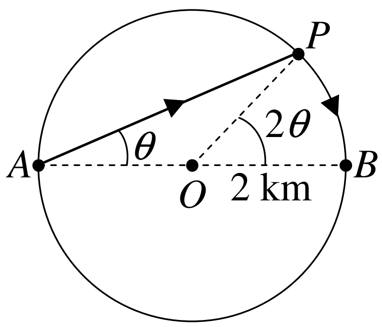
(i) Find the value of $\alpha + \beta + \gamma$. **1**

(ii) Find the value of $\alpha\beta\gamma$. **1**

(iii) Two of the roots are equal in magnitude but opposite in sign. **2**

Find the third root and hence find the value of k.

(c)



The diagram shows a circular lake, centre O, of radius 2 km with diameter AB. Pat can row at 3 km/h and can walk at 4 km/h and wishes to travel from A to B as quickly as possible. Pat considers the strategy of rowing direct from A to a point P and then walking around the edge of the lake to B. Let $\angle PAB = \theta$ radians, and let the time taken for Pat to travel from A to B by this route be T hours.

(i) Show that $T = \dfrac{1}{3}(4cos\theta + 3\theta)$. **2**

(ii) The value of θ for which $\dfrac{dT}{d\theta} = 0$ is of the form $\sin^{-1} a$. Find a and hence find, *to the nearest minute*, the corresponding time for this value of θ. **2**

(iii) If $\theta = 0$, find, *to the nearest minute*, the time taken and interpret your answer. **2**

(iv) If $\theta = \dfrac{\pi}{2}$, find, *to the nearest minute*, the time taken and interpret your answer. **2**

(v) Hence, determine what strategy Pat should employ to minimise the time taken to travel from A to B. **1**

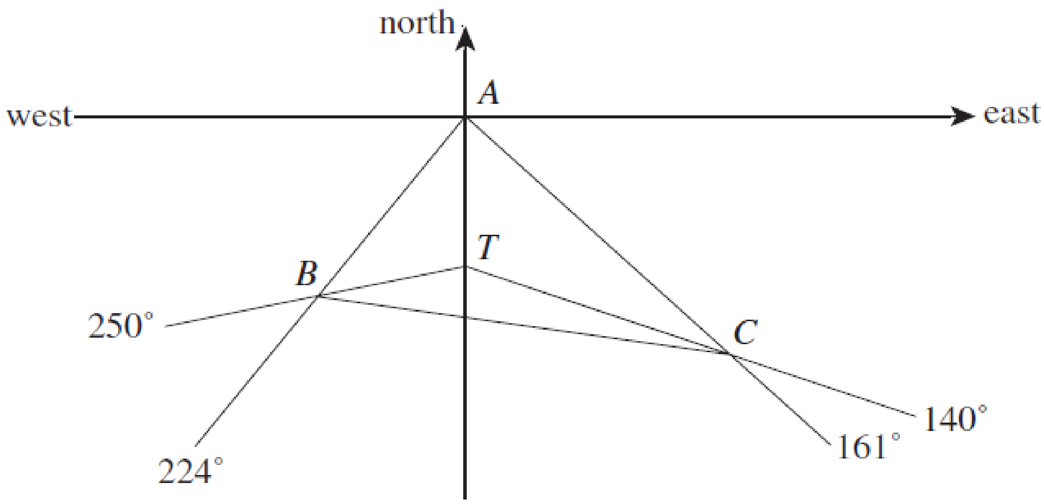
Question 14 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) A total of five players are selected at random from four sporting teams. Each of the teams consists of ten players numbered 1 to 10.

(i) What is the probability that of the five selected players, three are numbered '6' and two are numbered '8'? *Leave your answer as a fraction in lowest terms.* **1**

(ii) What is the probability that the five selected players contain at least four players from the same team? *Leave your answer as a fraction in lowest terms.* **2**

(b) From point A, the bearings of B and C are 224° and 161° respectively. From point T, 30 km due south of A, the bearings to B and C are 250° and 140° respectively.



<u>Copy</u> the diagram into your writing booklet.

(i) Show that the distance from B to C in kilometres is given by **3**

$$(BC)^2 = 900\left[\left(\frac{\sin 44^\circ}{\sin 26^\circ}\right)^2 + \left(\frac{\sin 19^\circ}{\sin 21^\circ}\right)^2 - \frac{2\sin 44^\circ \sin 19^\circ \cos 110^\circ}{\sin 26^\circ \sin 21^\circ}\right].$$

(ii) Hence or otherwise determine the time it will take to sail from B to C at an average speed of 10 km/h. *Give your answer to the nearest minute.* **2**

Question 14 continues on page 9

(c) (i) Prove by mathematical induction that for all counting numbers n with $n \geq 3$, **3**

$$\sum_{r=2}^{n-1} {}^{r}C_2 = {}^{n}C_3.$$

(ii) Show that $\sum_{r=1}^{n} (1+x)^{r-1} = \sum_{r=1}^{n} {}^{n}C_r x^{r-1}$. **2**

(iii) Using the result from (ii) alone show once more that for $n \geq 3$, $\sum_{r=2}^{n-1} {}^{r}C_2 = {}^{n}C_3$. **2**

(*Do not use induction.*)

End of paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \ , \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

2012 Year 12 Mathematics Extension 1 TRIAL SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

MULTIPLE CHOICE Q1-10

Q1 D   Q5 C   Q9 C

Q2 D   Q6 A   Q10 C

Q3 B   Q7 A

Q4 B   Q8 D

Question 11

a) $5x(x-2) \geq 3(x-2)^2$ ✓ $x \neq 2$

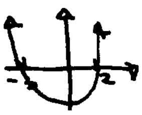
$5x^2-10x \geq 3(x^2-4x+4)$

$5x^2-10x \geq 3x^2-12x+12$

$2x^2+2x-12 \geq 0$

$x^2+x-6 \geq 0$

$(x+3)(x-2) \geq 0$



$x \leq -3$ ✓ or $x \geq 2$ ✓

b) $g(x) = x^2 - \log_e(x+1)$

(i) $g(0.7) = (0.7)^2 - \log_e(0.7+1)$
$= -0.04\ldots$ ✓

$g(0.9) = (0.9)^2 - \log_e(0.9+1)$
$= 0.168\ldots$

Since sign change $\therefore$ zero between 0.7 and 0.9, noting g is continuous.

Comments: establishing sign change. Don't deduct mark if discussion on continuity omitted.

(ii) $g(0.8) = 0.8^2 - \log_e(0.8+1)$
$= 0.05221\ldots > 0$

$g(0.7) < 0$ ✓

$\therefore$ zero between 0.7 and 0.8

$g(0.75) = 0.75^2 - \log_e(0.75+1)$
$= 0.002888\ldots > 0$

zero between 0.75 and 0.7

$\therefore \underline{\underline{0.7}}$ (1dp) ✓

Comments: for some progress. (Sign diagram also gains mark for working)

(11c next)

11d) $y = 2\sin x \quad y^2 = 4\sin^2 x$

$V = \pi \int_0^{\pi/4} 4\sin^2 x \, dx$

$= 4\pi \int_0^{\pi/4} \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$

$= 4\pi \left[\frac{x}{2} - \frac{1}{4}\sin 2x\right]_0^{\pi/4}$ ✓ for correct integration.

$= 4\pi\left[\left(\frac{\pi}{8} - \frac{1}{4}\sin\frac{\pi}{2}\right) - 0\right]$

$= \frac{4\pi^2}{8} - \frac{4\pi}{4}$

$= \left(\frac{\pi^2}{2} - \pi\right) u^3$ ✓ $\pi\left(\frac{\pi}{2} - 1\right) u^3$

11 c) Binomial Expansion of $\left(4x^3 - \frac{1}{x}\right)^{12}$

$= \sum_{r=0}^{12} {}^{12}C_r \, (4x^3)^{12-r} (-x^{-1})^r$

$= \sum_{r=0}^{12} {}^{12}C_r \, 4^{12-r} x^{36-3r} (-1)^r . (x^{-r})$

& require $x^{36-3r} . x^{-r} = 0$

$\Rightarrow 36 - 3r - r = 0$

$\Rightarrow 36 - 4r = 0$

$r = 9$ ✓

$\therefore {}^{12}C_9 \, 4^3 (-1)^9$ or -14080 ✓ F.T marks awarded if r is incorrect.

11e) i) $T = 30 + Ae^{-kt}$

$Ae^{-kt} = T - 30$

$dT/dt = -kAe^{-kt}$ ✓ must show

$= -k(T-30)$

ii) $250 = 30 + A$ ✓ must show

$A = 220$

iii) $150 = 30 + 220e^{-20k}$

$\therefore k = 0.03$ (2dp) ✓

iv) $80 = 30 + 220e^{-0.03t}$ ✓

$t = 49.386$

$= 49$ minutes ✓

(49 minutes 23 seconds)

2012 Year 12 Mathematics Extension 1 TRIAL SOLUTIONS

Suggested Solution (s)	Comments
Question 12	
ai) $P(\text{success}) = 0.7$ $P(\text{not success}) = 0.3$ ${}^{10}C_9\,(0.7)^9(0.3)^1 = \underline{12\%}$ ✓	
ii) $n \times p = 10 \times 0.7 = \underline{7 \text{ people}}$ ✓	
b) $P(5) = 0$ $125 + 5a + b = 0$ — ① $P(-5) = -60$ $-125 - 5a + b = -60$ — ② Solve simultaneously ① + ② $5a + b + 125 = 0$ $-5a + b - 65 = 0$ + $2b + 60 = 0$ $b = -30$ ✓ $\therefore \quad a = -19$ ✓	✓ meaningful progress
c) let $u = \tan x$ $\frac{du}{dx} = \sec^2 x$ $du = \sec^2 x\,dx$ at $x = 0$ $u = 0$ $x = \pi/3$ $u = \sqrt{3}$ $\int_0^{\sqrt{3}} u^2\,du$ ✓ $= \left[\frac{u^3}{3}\right]_0^{\sqrt{3}}$ ✓ $= \frac{(\sqrt{3})^3}{3} - 0$ $= \underline{\sqrt{3}}$ ✓	Allow for $\int_\alpha^\beta u^2\,du = \left[\frac{u^3}{3}\right]_\alpha^\beta$ $= \left[\frac{\tan^3 x}{3}\right]_0^{\pi/3}$ $= \sqrt{3}$ upon re-instating the original limits Preamble: Let α, β be the change of limits under the change of variable.
di) $\cos^2 3t = 3/4$ $\cos 3t = \pm\sqrt{3/4}$ $= \pm\sqrt{3}/2$ $3t = \pi/6, 5\pi/6 \ldots$ $t = \pi/18, 5\pi/18 \ldots$ $\therefore$ at $t = \underline{\pi/18}$ seconds ✓	
dii) $v = dx/dt = 2\cos 3t(-3\sin 3t)$ $= -6\cos 3t \sin 3t$ $= -3 \times 2\cos 3t \sin 3t$ $= -3 \times \sin 6t$ (as $\sin 2\theta = 2\sin\theta\cos\theta$) at $t = \pi/18$ $dx/dt = -3\sin \pi/3$ < 0 since $v < 0$ the particle is travelling to the left towards the origin (or in the negative direction)	Evidence of calculation of $v\left(\frac{\pi}{18}\right)$ required. ✓ reason ✓ answer
diii) $\frac{dx}{dt} = -3\sin 6t$ $d^2x/dt^2 = -3\cos 6t \times 6$ ✓ $= -18\cos 6t$ $= -18(2\cos^2 3t - 1)$ (as $\cos 2\theta = 2\cos^2\theta - 1$) $= -18(2x - 1)$ ✓	Equivalently $\ddot{x} = \frac{d}{dt}(-6\cos 3t \sin 3t)$ $= -18\cos^2 3t + 18\sin^2 3t$ $= -18(\cos^2 3t - \sin^2 3t)$ $= -18(2\cos^2 3t - 1)$ $= -18(2x - 1)$
div) $a = -18(2x - 1)$ $= -36(x - \frac{1}{2})$ $= -6^2(x - \frac{1}{2})$ ✓ which is in the form of $\ddot{x} = -n^2(x - b)$ $\therefore$ it is undergoing S.H.M. with centre of oscillation at $x = \frac{1}{2}$	Equivalently $\ddot{x} = -n^2(x - x_0)$
dv) Period $= \frac{2\pi}{n}$ $= 2\pi/6$ $= \underline{\pi/3}$ seconds ✓	

2012 Year 12 Mathematics Extension 1 TRIAL SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Question 13

a) ${}^{12}C_2 \times {}^{25}C_4 = \underline{\underline{834\,900}}$ ✓

b i) $\alpha+\beta+\gamma = -b/a$

$= 5/2$ ✓

Sum of roots

ii) $\alpha\beta\gamma = -d/a$

$= -40/2$

$= -20$ ✓

Product of Roots

iii) $\alpha + (-\alpha) + \gamma = 5/2$

$\underline{\underline{\gamma = 5/2}}$ ✓ — ①

$\cdot\,\beta = -\alpha$ with γ the third root

$\alpha(-\alpha)\gamma = -20$

$\Rightarrow \quad -\alpha^2 = -8$ using ①

or $\alpha^2 = 8$ — ②

Also $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{K}{2}$ & $\beta = -\alpha$

Sum of roots in product pairs

$\Rightarrow \quad -\alpha^2 + \alpha\gamma - \alpha\gamma = K/2$

$\Rightarrow \quad -\alpha^2 = K/2$

or $\quad K = -2\alpha^2$

From ② $\alpha^2 = 8$

$\therefore \underline{\underline{K = -16}}$ ✓

c i) $\angle APB = \pi/2$ ($\angle$ in a semicircle $= 90°$)

$\therefore \cos\theta = \frac{AP}{AB}$

$AP = AB\cos\theta = 4\cos\theta$ ✓ some meaningful progress

· Cosine Rule also available on $\triangle OAP$.

arc length $= r\theta$

arc $PB = OB \times 2\theta$

$= 2 \times 2\theta$

$= 4\theta$

Time from A to B = Time for AP + Time for PB

$S = D/T$

$T = D/S$

$= \frac{AP}{3} + \frac{PB}{4}$ ✓

$= \frac{4\cos\theta}{3} + \theta$

$= \frac{1}{3}(4\cos\theta + 3\theta)$ as required

(ii) $dT/d\theta = \frac{1}{3}(-4\sin\theta + 3)$

$\frac{1}{3}(-4\sin\theta + 3) = 0$

$\sin\theta = \frac{3}{4}$

So $a = \frac{3}{4}$ ✓

$\theta = \sin^{-1}(\frac{3}{4})$

$\Rightarrow \cos\theta = \frac{\sqrt{7}}{4}$ as θ is acute.

$T = \frac{1}{3}(4\cos(\sin^{-1}\frac{3}{4}) + 3 \times (\sin^{-1}\frac{3}{4}))$

$= 1$ hour 44 minutes ✓

(iii) $T = \frac{1}{3}(4\cos 0 + 3 \times 0)$

$= 1$ hour 20 minutes ✓

it would take this long to row directly to B ✓

iv) $T = \frac{1}{3}(4\cos\frac{\pi}{2} + 3 \times \frac{\pi}{2})$

$= 1$ hour 34 minutes ✓

it would take this long to walk around the lake ✓

v) Pat should row directly across the lake to B ✓

2012 Year 12 Mathematics Extension 1 TRIAL SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Question 14

ai) 4 players numbered '6', 4 players numbered '8' from a total of 40 players

Three 6's selected ${}^4C_3$ ways

Two 8's selected ${}^4C_2$ ways.

Five players selected ${}^{40}C_5$ ways

$\therefore \dfrac{{}^4C_3 \times {}^4C_2}{{}^{40}C_5} = \dfrac{1}{27417}$ ✓

Comments: Unsimplified: $\dfrac{24}{658{,}008}$ No penalty if fraction unsimplified

aii) "At least 4 players" = 4 or 5 players

5 players from 1 team selected ${}^{10}C_5$ ways

But there are 4 teams, hence 5 players from the same team can be selected ${}^4C_1 \times {}^{10}C_5$ ways

4 players from one team and one player from the remaining teams (30 players) can be selected ${}^4C_1 \times {}^{10}C_4 \times {}^{30}C_1$ ways.

Allowance for 4 teams; 4 members from 1 team; Remaining member

$\therefore \dfrac{{}^4C_1 \times {}^{10}C_5}{{}^{40}C_5} + \dfrac{{}^4C_1 \times {}^{10}C_4 \times {}^{30}C_1}{{}^{40}C_5}$ ✓

$= \dfrac{28}{703}$ ✓

Comments: Some meaningful progress. Unsimplified: $\dfrac{26\,208}{658{,}008}$ No penalty if fraction unsimplified.

bi)

In $\triangle ABT$: $\dfrac{BT}{\sin 44^\circ} = \dfrac{30}{\sin 26^\circ}$

$BT = \dfrac{30 \sin 44^\circ}{\sin 26^\circ}$

In $\triangle ATC$ $\dfrac{TC}{\sin 19^\circ} = \dfrac{30}{\sin 21^\circ}$

$TC = \dfrac{30 \sin 19^\circ}{\sin 21^\circ}$

Comments: ✓ either BT or TC correct

Using the cosine rule in $\triangle BTC$

$(BC)^2 = (BT)^2 + (TC)^2 - 2(BT)(TC)\cos(BTC)$ ✓

$= 30^2\left(\dfrac{\sin 44}{\sin 26^\circ}\right)^2 + 30^2\left(\dfrac{\sin 19^\circ}{\sin 21^\circ}\right)^2$ ✓

$- 2 \times \dfrac{30 \sin 44^\circ}{\sin 26^\circ} \times \dfrac{30 \sin 19^\circ}{\sin 21^\circ} \times \cos 110^\circ$

$= 900\left[\left(\dfrac{\sin 44^\circ}{\sin 26^\circ}\right)^2 + \left(\dfrac{\sin 19^\circ}{\sin 21^\circ}\right)^2 - 2\dfrac{\sin 44 \sin 19^\circ \cos 110}{\sin 26^\circ \sin 21^\circ}\right]$

bii) $(30 \times 2.078735\ldots)$ unrounded.

$BC = 30 \times 2.08$

$= 62.36$ km ✓ (62.362054) unrounded

$D = s \times t$

At $s = 10$ km/h trip takes

6.236 hours (6.2362054) unrounded

(6 hours 14 minutes) ✓

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Q14

c i) Let $P(n)$ be the proposition: $\sum_{r=2}^{n-1} {}^{r}C_2 = {}^{n}C_3$, $n \geq 3$.

Then $LHS_{P(3)} = \sum_{r=2}^{3-1} {}^{r}C_2$

$= \sum_{r=2}^{2} {}^{r}C_2$

$= {}^{2}C_2$

$= 1$

$= {}^{3}C_3$

$= RHS_{P(3)}$.

So $P(3)$ is true. ✓

Assume the proposition is true for some arbitrary counting no. K $(K \geq 3)$, i.e., assume :- $\sum_{r=2}^{K-1} {}^{r}C_2 = {}^{K}C_3$. Our aim is to use this assumption to show that $P(K+1)$ is necessarily true.

$LHS_{P(K+1)} = \sum_{r=2}^{K} {}^{r}C_2$

$= \left(\sum_{r=2}^{K-1} {}^{r}C_2\right) + {}^{K}C_2$ — Partitioning summation

$= {}^{K}C_3 + {}^{K}C_2$ ⊛ using induction hypothesis — Equivalently, by assumption

$= {}^{K+1}C_3$ using PASCAL's TRIANGLE RELATION — a.k.a. as the Addition Property for Binomial Coeff's

$= RHS_{P(K+1)}$.

✓✓ process, 1/2 for meaningful progress

So $P(K) \Rightarrow P(K+1)$. As $P(3)$ is true we conclude that $P(n)$ is true for all counting nos. $n \geq 3$ by the PMI.

⊛ Using 'brute force', i.e., factorials:-

$${}^{K}C_3 + {}^{K}C_2 = \frac{K!}{3!\,(K-3)!} + \frac{K!}{2!\,(K-2)!}$$

$$= \frac{K!\,(K-2)}{3!\,(K-2)!} + \frac{K! \times 3}{3!\,(K-2)!}$$

$$= \frac{K!\,[(K-2)+3]}{3!\,(K-2)!} = \frac{(K+1)!}{3!\,(K-2)!} = {}^{K+1}C_3.$$

c ii)

Firstly, $\sum_{r=1}^{n} {}^{n}C_r x^{r-1}$

$= {}^{n}C_1 + {}^{n}C_2 x + {}^{n}C_3 x^2 + \dots + {}^{n}C_n x^{n-1}$.

$\sum_{r=1}^{n} (1+x)^{r-1}$

$= (1+x)^0 + (1+x) + (1+x)^2 + \dots + (1+x)^{n-1}$ — Geometric Series with n terms. $S_n = \frac{a(r^n-1)}{r-1}$.

$= \frac{1 \times ((1+x)^n - 1)}{(1+x)-1} = \frac{1}{x}\left[(1+x)^n - 1\right]$

$= \frac{1}{x}\left[\underbrace{{}^{n}C_0 + {}^{n}C_1 x + {}^{n}C_2 x^2 + \dots + {}^{n}C_n x^n}_{\text{Binomial Expansion of } (1+x)^n} - 1\right]$ ✓

$= \frac{1}{x}\left[{}^{n}C_1 x + {}^{n}C_2 x^2 + \dots + {}^{n}C_n x^n\right]$ — ${}^{n}C_0 = 1$

$= {}^{n}C_1 + {}^{n}C_2 x + \dots + {}^{n}C_n x^{n-1}$ ✓

$= \sum_{r=1}^{n} {}^{n}C_r x^{r-1}$, as required.

1/2 for meaningful progress.

c iii) The coefficient of x^2 in $\frac{1}{x}\left[(1+x)^n - 1\right]$ is ${}^{n}C_3$. Equivalently, from $(1+x)^0 + (1+x)^1 + (1+x)^2 + (1+x)^3 + (1+x)^4 + (1+x)^5 + \dots + (1+x)^{n-1}$, the coefficient of x^2 is :-

Division by x requires the extraction of the x^3 term in the expansion of $(1+x)^n$.

${}^{2}C_2 + {}^{3}C_2 + {}^{4}C_2 + {}^{5}C_2 + \dots + {}^{n-1}C_2$.

So, ${}^{2}C_2 + {}^{3}C_2 + {}^{4}C_2 + {}^{5}C_2 + \dots + {}^{n-1}C_2 = {}^{n}C_3$

i.e. $\sum_{r=2}^{n-1} {}^{r}C_2 = {}^{n}C_3$, as required

✓✓ for process. 1/2 for recognition that coeff't of x^2 sought plus partial success in algebraic extraction

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2012 Year 12 Mathematics Extension 1 TRIAL SOLUTIONS

Suggested Solution (s)	Comments
MULTIPLE CHOICE SOLUTIONS:	1. D 2. D 3. B 4. B 5. C 6. A 7. A 8. D 9. C 10. C
Question 1: EVENT SPACE A(say) is:- $A=\{(1,1);(2,1);(1,2);(1,3);(2,2);(3,1);(1,4),(2,3),(3,2),(4,1)\}$ & $n(A)=10$ $\therefore P(\text{TOTAL}<6)=\frac{10}{36}=\frac{5}{18}$. ∴ (D)	
Question 2: Domain satisfies $-1\le\frac{5x}{2}\le 1$ $\Rightarrow -\frac{2}{5}\le x\le\frac{2}{5}$ & range is $0\le y\le\pi$. So (D)	
Question 3: 2L, 3O, 2N, 14 letters. # of permutations $=\frac{14!}{2!3!2!}=\frac{14!}{4!}$ ∴ (B)	$2!3!2!=24$
Question 4: $m_1=2$ & $m_2=\frac{1}{3}$ with tan(acute angle) $=\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|=\left\|\frac{2-1/3}{1+(2)(\frac{1}{3})}\right\|=\left\|\frac{5/3}{5/3}\right\|$ i.e. $\tan(\text{acute}\ \angle)=1$ $\Rightarrow$ acute $\angle=45^\circ$ So (B)	
Question 5: Use host as reference marker. There are 5 choices then for the hostess & 6! for the guests. All up, $5\times 6!=3600$ different seating arrangements. So (C)	
Question 6: For x large, $\frac{2x^3+4x}{x^2-4}\approx\frac{2x^3}{x^2}=2x$ & $2x\to\infty$ as $x\to\infty$. So (A)	• Dominant terms • Other alternatives generate finite values.
Question 7: $\frac{dt}{dx}=\frac{1}{2}\sec^2\frac{x}{2}$ $\Rightarrow\frac{dt}{dx}=\frac{1}{2}\left(1+\tan^2\frac{x}{2}\right)$ i.e. $\frac{dt}{dx}=\frac{1}{2}(1+t^2)$ $\Rightarrow\frac{dx}{dt}=\frac{2}{1+t^2}$ ∴ (A)	. chain rule . trig identity . $\frac{dx}{dt}\cdot\frac{dt}{dx}=1$ (Inverse function thm)
Question 8: For $\int\frac{2x}{\sqrt{1+x^2}}dx$, let $u=1+x^2$ $\Rightarrow du=2x\,dx$ & integral becomes $\int\frac{du}{\sqrt{u}}=2\sqrt{u}+C$ i.e. $\int\frac{2x\,dx}{\sqrt{1+x^2}}=2\sqrt{1+x^2}+C$ ∴ (D)	
Question 9: $\frac{dx}{dt}=0.5$ units/s & $\frac{dy}{dx}=6x^2=m$ We require $\frac{dm}{dt}=\frac{dm}{dx}\cdot\frac{dx}{dt}$ $=12x\times 0.5$ $=6x$ $=6$ when $x=1$ ∴ (C)	Gradient function $\frac{dm}{dx}$ measured in units^{-1} & $\frac{dx}{dt}$ measured in units.s^{-1}.
Question 10: $\tan\frac{x}{2}$ is undefined when $\frac{x}{2}=\dots,\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\dots$ $\Rightarrow x=\dots,\pi,3\pi,5\pi,\dots$ ∴ (C)	Domain of tan f^n is $x\ne(2k+1)\frac{\pi}{2}$, $k\in\mathbb{Z}$.